PERIDYNAMIC THEORY FULLY-COUPLED THERMAL AND DEFORMATION FIELDS

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Abstract

Prediction of crack propagation path is especially a challenging due to the interaction of different fields such as thermal diffusion and deformation. For a fully coupled thermomechanical analysis, the deformation field is influenced by the thermal influence coming in via a thermomechanical constitutive model. The heat transfer facet is governed by the thermal energy equation with the structural heating and cooling term present in the thermal energy equation. The classical governing equations involve the spatial derivatives which are not defined at discontinuities such as a crack. Also, the solutions to the governing equations of thermal and deformation fields are constructed in two different computational domains; thus, the effect of crack growth is not immediately reflected in the domain of thermal field without rediscretization. Furthermore, prediction of crack growth in general is achieved through external criteria, post processing and not intrinsic to the governing equations. The peridynamic (PD) theory introduced by Silling (2000) permits the crack growth analysis in the same computational domain for both thermal and deformation fields. The PD theory was originally applied to investigate crack growth without the need for external criteria.

Gerstle et al. (2008) extended original PD formulation to include the effect of thermal and electrical fields in a one-dimensional body through a decoupled analysis. Later, Bobaru and Duangpanya (2010) developed a one-dimensional PD heat conduction equation. They also solved two-dimensional heat conduction problems with discontinuities, (Bobaru and Duangpanya, 2012). As an extension to the decoupled (sequential) thermo-mechanical model of Kilic and Madenci (2010), Agwai (2011) developed a one-dimensional fully-coupled PD approach.

In this study, the fully-coupled peridynamic approach of Agwai is extended to perform transient thermo-mechanical analysis in two-dimensional finite domains with a pre-existing crack. Numerical implementations are presented and compared with the previous studies. The effect of thermomechanical cooling is discussed. No work has been published on the crack propagation including the effect of thermo mechanical loading within the PD framework.

The degree of thermomechanical coupling depends on the material. Typically, the strength of coupling is measured via the nondimensional quantity known as the coupling coefficient defined as

$$\epsilon = \frac{\beta_{cl}^2 T_o}{\rho\, c_v (\lambda + 2\mu)} \tag{1}$$

for which $\beta_{cl} = (3\lambda + 2\mu)\alpha$, E is the elastic modulus, α is the coefficient of thermal expansion, ρ is the mass density, c_v is the specific heat capacity, λ and μ are Lamé's constants and T_o is the reference temperature at which the stress in the body is zero (Nowinski, 1978). The coupling coefficient for metals is significantly lower than those of plastics.

The fully coupled bond-based thermomechanical equations derived by Agwai (2011) are given by

$$\rho\ddot{\mathbf{u}} = \int_H \hat{\mathbf{f}}\, dV_{\mathbf{x}'} + \mathbf{b} \quad \text{where} \quad \hat{\mathbf{f}} = \frac{\boldsymbol{\xi}+\boldsymbol{\eta}}{\left|\boldsymbol{\xi}+\boldsymbol{\eta}\right|} c\left(s - \alpha\theta_{avg}\right)$$

$$\text{and} \tag{2}$$

$$\rho c_v \dot{T} = \int_H \left(\kappa \frac{\tau}{\left|\boldsymbol{\xi}\right|} - T_o \frac{c}{2} \alpha \dot{e} \right) dV_{\mathbf{x}'} + q_b$$

where the first equation is the conversation of linear momentum (i.e. the structural equation of motion) with a thermoelastic constitutive relation given by the response function. The second equation is the conversation of thermal energy (i.e. the heat transfer equation) with a contribution from deformational heating and cooling. In the first equations, $\boldsymbol{\xi} = \mathbf{x}' - \mathbf{x}$ is the initial relative position, $\boldsymbol{\eta} = \mathbf{u}' - \mathbf{u}$ is the relative displacement, *s* is the stretch, *c* is the bond constant, and α is the coefficient of thermal expansion. The average temperature, θ_{avg} is given by

$$\theta_{avg} = \frac{\left(T\left(\mathbf{x}\right) - T_o\right) + \left(T\left(\mathbf{x}'\right) - T_o\right)}{2}. \tag{3}$$

In the second equation, κ is the microconductivity, τ is the temperature difference between $\mathbf{x}$ and $\mathbf{x}'$, and c_v is the specific heat capacity. The reference temperature is T_o. The bond extension is defined by

$$e = \left|\boldsymbol{\eta} + \boldsymbol{\xi}\right| - \left|\boldsymbol{\xi}\right| \tag{4}$$

and the time rate of change of the bond extension becomes

$$\dot{e} = \frac{\boldsymbol{\eta} + \boldsymbol{\xi}}{\left|\boldsymbol{\eta} + \boldsymbol{\xi}\right|} \cdot \dot{\boldsymbol{\eta}} \tag{5}$$

where $\dot{\boldsymbol{\eta}}$ is the time rate of change of the relative displacement.

For the numerical treatment of the fully coupled thermoelastic peridynamic system of equations, the system is partitioned naturally according to the structural and thermal fields; thus, the equation of motion is solved for the displacement field, and the heat transfer equation is solved for the temperature field. Explicit time stepping schemes are utilized to approximate the solutions to both equations.

This solution method is verified by solving a problem previously considered by Tehrani and Eslami (2000) by using Boundary Element Method. It concerns a square plate of isotropic material, as shown in Fig.1, under a pressure shock loading. The non-dimensional length and width of the square plate is 10 and it has a unit thickness. The pressure shock is applied at $x = 0$

in the positive x direction and thermo-mechanical equations are both solved for uncoupled $\epsilon = 0$ and coupled cases $\epsilon \neq 0$. The mechanical and thermal boundary conditions are

$$
\begin{aligned}
& u_x\left(x=10, y, t\right) = u_y\left(x=10, y, t\right) = 0 \\
& T_{,x}\left(x=10, y, t\right) = 0 \\
& \sigma_{yy}\left(x, y=\pm 5, t\right) = \sigma_{xy}\left(x, y=\pm 5, t\right) = 0 \\
& T_{,y}\left(x, y=\pm 5, t\right) = 0 \\
& \sigma_{xx}(x=0, y, t) = 5t\exp(-2t)
\end{aligned}
\tag{6}
$$

where t is nondimension time. For the peridynamic model, a grid spacing of $dx = dy = 0.02$ and a time step of $dt = 5\times 10^{-4}$ are used.

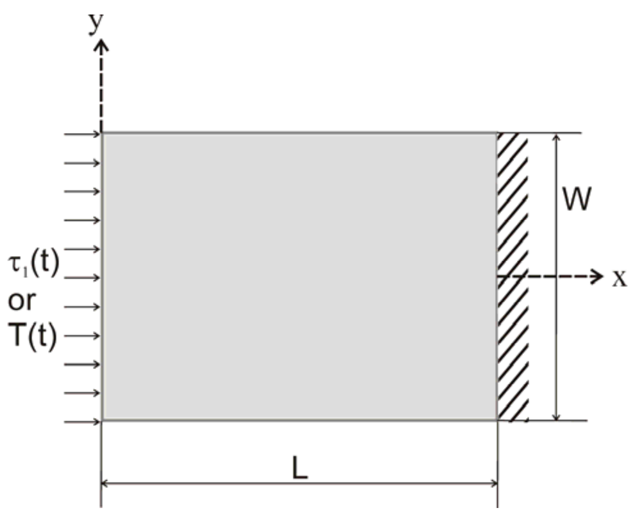

Figure 1. Geometry and boundary conditions of the plate under pressure shock.

Fig. 2 shows the temperature distribution due to the mechanical loading at time 3 and 6. When the coupling coefficient, ϵ, is zero, no temperature change is expected. However, when the coupled effect is included, even though mechanical loading is applied, temperature change is expected. The compressive stress along the boundary causes a temperature rise. As observed in this figure, the peak of the temperature distribution moves to right as time progresses. Very similar behavior is also observed by Tehrani and Eslami (2000). Fig. 3 shows the axial displacement along the x-axis. The PD results are also in close agreement with the BEM results, (Tehrani and Eslami, 2000).

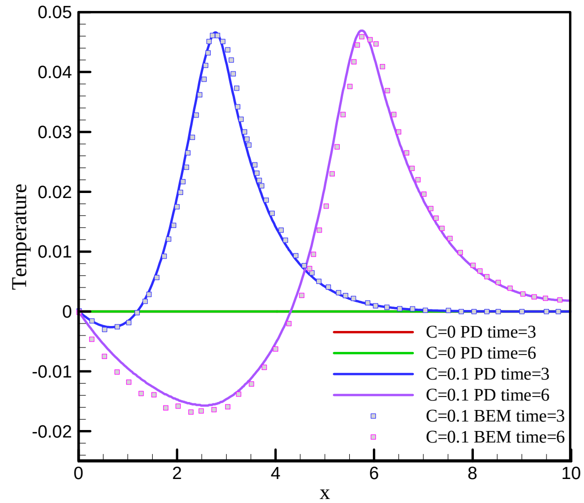

Figure 2. Comparison of temperature along the middle of the plate for uncoupled and coupled cases.

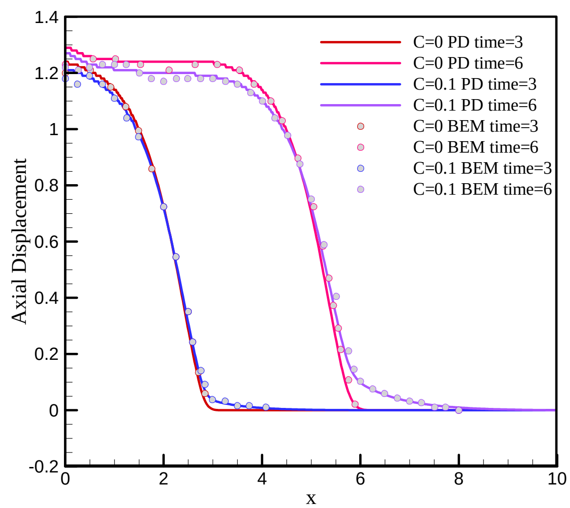

Figure 3. Comparison of axial displacement along the middle of the plate for uncoupled and coupled cases

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