Noname manuscript No.
(will be inserted by the editor)

Analysis of the volume-constrained peridynamic Navier equation of linear elasticity

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Received: date / Accepted: date

Abstract Well-posedness results for the state-based peridynamic nonlocal continuum model of solid mechanics are established with the help of a nonlocal vector calculus. The peridynamic strain energy density for an elastic constitutively linear anisotropic heterogenous solid is expressed in terms of the field operators of that calculus, after which a variational principle for the equilibrium state is defined. The peridynamic Navier equilibrium equation is then derived as the first-order necessary conditions and are shown to reduce, for the case of homogeneous materials, to the classical Navier equation as the extent of nonlocal interactions vanishes. Then, for certain peridynamic constitutive relations, the peridynamic energy space is shown to be equivalent to the space of square-integrable functions; this result leads to well-posedness results for volume-constrained problems of both the Dirichlet and Neumann types. Using standard results, well-posedness is also established for the time-dependent peridynamic equation of motion.

Keywords peridynamic theory, Navier equation, nonlocal operators, vector calculus, volume-constrained problems

Mathematics Subject Classification (2000) 35B40, 35J20, 35J25, 35Q99, 45A05, 45K05, 74B05

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