Optimal distributed control of nonlocal steady diffusion problems

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A control problem constrained by a nonlocal steady diffusion equation that arises in several applications is studied. The control is the right-hand side forcing function and the objective of control is a standard matching functional. A recently developed nonlocal vector calculus is exploited to define a weak formulation of the state system. When sufficient conditions on certain kernel functions and the volume constraints hold, the existence and uniqueness of the optimal state and control is demonstrated and an optimality system is derived. We demonstrate the convergence, as the nonlocal interactions vanish, of the optimal nonlocal state to the optimal state of a local PDE-constrained control problem. We also define continuous and discontinuous Galerkin finite element discretizations of the optimality system for which we derive a priori error estimates. Numerical examples are provided illustrating these convergence results and also illustrating the differences between optimal controls and states obtained for the nonlocal diffusion equations and for PDEs. In particular, we observe that using nonlocal models result in a better match to non-smooth target functions and a reduced cost of control.