A Nonlocal Vector Calculus with Applications to Diffusion and Mechanics

A review of a newly developed nonlocal calculus is given, including definitions of nonlocal divergence, gradient, and curl operators and derivations of a nonlocal Gauss theorem and Green's identities. Through appropriate limiting processes, relations between the nonlocal operators and their differential counterparts are established. Nonlocal problems analogous to boundary-value problems for partial differential equations are defined using the operators from the nonlocal calculus. The nonlocal calculus is used to define weak formulations of the nonlocal diffusion and mechanics problems which are then analyzed, showing, for example, that unlike elliptic partial differential equations, these problems do not necessarily result in the smoothing of data. We briefly consider finite element methods for nonlocal problems, focusing on solutions containing jump discontinuities; in this setting, discontinuous Galerkin methods are conforming and can lead to optimally accurate approximations.

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