Spatial Randomness in Local and Non-Local Material Mechanics Models

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Turning continuum mechanics models into stochastic theories is necessary in many applications and here we systematically examine what can and cannot be done in local and non-local solid mechanics. First, we recognize that the dependent quantities (displacement, velocity, stress, ...) as well as constitutive responses (stiffness, viscosity, ...) are tensor random fields (TRF). We next focus on the basic properties of TRFs of a wide-sense stationary (WWS) kind, and, especially, on isotropic correlation functions with locally isotropic realizations (WWSI). Using the theory of invariants, we examine the consequences of field equations (balance laws, strain-displacement relations) for stress and strain fields, rotation, curvature-torsion, and couple-stress TRFs in stochastic classical and micropolar theories, which, in turn, have implications for micro-heterogeneous structure of many materials. Secondly, we examine what TRFs are admissible if they are to represent constitutive responses. Now, the key role is played by the positive definiteness of response tensors in combination with the Hill-Mandel condition which has to be chosen according to a type of material (elastic versus inelastic) and type of loading (static, incremental, relaxational, or dynamic …). An obvious application of all these models pertains to (i) stochastic (SFE) rather than deterministic finite elements and (ii) wavefronts in random media.

Upon combination of the above results, we find that a locally isotropic anti-plane stiffness TRF of WSSI kind cannot be chosen arbitrarily, but its anisotropy has to be admitted. By virtue of mathematical analogies this result carries over to a number of other physical problems: thermal conductivity, electrical conduction, magnetostatics, diffusion, etc. This provides a stepping-stone to 4th rank stiffness TRF: in classical linear elasticity one may not adopt two Lamé constants as random fields with smoothly inhomogeneous realizations. Incidentally, taking only Young's modulus as a random field with Poisson's ratio constant – as practiced in conventional SFE methods – is completely unphysical. We then move from a local elastic continuum to micropolar elasticity, to nonlocal elasticity, and to bond-based-peridynamics. We end by considering wave propagation in peridynamics on random and/or fractal fields of material properties.